THE PROBLEM OF A DIHEDRAL PISTON

V. M. Teshukov

The problem of the plane nonstationary motion of a gas behind a dihedral piston is considered. The problem is linearized on the assumption that the piston angle α is small. The mixed problems and the Goursat problem are solved for the linearized double-wave equation in the region of hyperbolicity and then the mixed boundary value problem is solved in the region of ellipticity. The solutions are obtained in elementary functions and quadratures.

The problem in question was investigated in the plane of the hodograph in [1] and [2]. In [1] the case in which the velocity of the piston is sufficiently large and the gas flows into a vacuum was considered. In [2] a numerical solution of the problem was obtained in the region of hyperbolicity of the double wave equation.

1. Formulation of the Problem and Basic Relations

At time t = 0 let a polytropic gas with equation of state $p = a^2\rho^\gamma$ (p is pressure, γ is the adiabatic exponent, a^2 = const) be at rest within the dihedral angle formed by two intersecting planes P_1 and P_2. We consider the problem of finding the nonstationary plane gas flow as the dihedral angle is withdrawn at an constant velocity U_0 directed along the bisectrix of the angle between the planes P_1 and P_2. It is assumed that this angle is less then π

We seek a solution in the class of conical flows. If it is assumed that there are no shock waves, these flows will be isentropic and potential. The unknown velocity components u, and v and the speed of sound c will depend on the two independent variables $\xi = x/t$, $\eta = y/t$, where x, and y are the Cartesian coordinates of the plane and t is time. At small piston angles α shock waves with an intensity of the order of α may occur, but in the linear approximation the motion may be assumed isentropic, since the entropy jump must be of order α^3.

We consider conical, irrotational, and isentropic flows of a polytropic gas. We introduce the independent variables $\xi = x/t$, $\eta = y/t$ and represent the unknown functions in the form

$$u = \xi + U(\xi, \eta), \quad v = \eta + V(\xi, \eta), \quad p = P(\xi, \eta), \quad \rho = R(\xi, n) \tag{1.1}$$

If we introduce the potential in the form

$$\varphi_\xi = U, \quad \varphi_\eta = V \tag{1.2}$$

the basic equations of two-dimensional gasdynamics lead to the following quasi-linear equation describing the potential conical flows:

$$(U^2 - C^2)(\varphi_{\xi\xi} + 1) + 2UV\varphi_{\xi\eta} + (V^2 - C^2)(\varphi_{\eta\eta} + 1) = 0 \tag{1.3}$$

where

$$C^2 = (1 - \gamma)[\varphi + {}^1/_2\,(U^2 + V^2)] \tag{1.4}$$

Relation (1.4) is an analog of the Bernoulli integral. We establish the coordinate system in the flow plane, directing the x-axis along the axis of piston symmetry and the y-axis at right angles to it, so that at time t = 0 the coordinate origin coincides with the vertex of the piston angle Q. The picture of motion in

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the plane $\xi\eta$ coincides with that in the xy-plane at time t = l (Fig. 1). At large $|\eta|$ the motion is an undisturbed simple Riemann wave

$$C = \left(\frac{2}{\gamma+1} C_0 + \frac{\gamma-1}{\gamma+1} \xi_1\right), \qquad U\cos\alpha - V\sin\alpha = -C \quad U\sin\alpha + V\cos\alpha = -\eta_1, \qquad \xi_1 \in M_1 \tag{1.5}$$

and a constant flow

$$\begin{gathered} C = C_1 = C_0 + {}^1/_2(\gamma-1)U_0\cos\alpha, \qquad U\cos\alpha - V\sin\alpha = U_0\cos\alpha - \xi_1 \\ U\sin\alpha + V\cos\alpha = -\eta_1, \qquad \xi_1 \in M_2 \\ (U_0 < 0, \ \xi_1 = \xi\cos\alpha - \eta\sin\alpha, \ \eta_1 = \xi\sin\alpha + \eta\cos\alpha, \\ M_1 = [C_0 + {}^1/_2(\gamma+1)U_0\cos\alpha, \ C_0], M_2 = [U_0\cos\alpha, \ C_0 + {}^1/_2(\gamma+1)U_0\cos\alpha] \end{gathered} \tag{1.6}$$

Here, C_0 is the speed of sound in the quiescent gas and U_0 is the piston velocity; by virtue of flow symmetry the equations are given for $\eta \geq 0$ only. Here and in what follows it is assumed that $\gamma \neq 3$; the solution for $\gamma = 3$ is similarly constructed.

A flow of the double-wave type is sought in the neighborhood of the vertex Q. The simple wave and the constant flow adjoin the double wave along the characteristics BE and BF starting from point B ($C_0 \sec\alpha$, 0). The equation of the characteristics for Eq. (1.3) has the form:

$$(U^2 - C^2)\, d\eta^2 - 2UV d\xi d\eta + (V^2 - C^2) d\xi^2 = 0 \tag{1.7}$$

Using the known values of U, V, and C along BE, we integrate Eq. (1.7); as a result we obtain the equation of segment BD

$$\eta_1^2 = -\frac{\gamma+1}{\gamma-3} C^2 + \left(\mathrm{tg}^2\alpha + \frac{\gamma+1}{\gamma-3}\right) C_0^{\frac{\gamma-3}{\gamma-1}} C^{\frac{\gamma+1}{\gamma-1}}, \qquad \xi_1 \in M_1 \tag{1.8}$$

and the equation of segment DE

$$\begin{gathered} \eta_1 = \frac{1}{T}\left[\left(\mathrm{tg}^2\alpha + \frac{\gamma+1}{\gamma-3}\right) C_0^{\frac{\gamma-3}{\gamma-1}} C_1^{\frac{2}{\gamma-1}} - 2\,\frac{\gamma-1}{\gamma-3} C_1\right](\xi_1 - U_0\cos\alpha - C_1) + T \\ \xi_1 \in M_2 \end{gathered} \tag{1.9}$$

where C is given in (1.5) and the constant T = η_1 at point D.

Thus, in exact formulation the problem reduces to finding in a known region the solution φ of Eq. (1.3), which takes the value φ_0 on the boundary characteristics BE and BF and satisfies the impermeability condition $\varphi_\xi n_1 + \varphi_\eta n_2 = 0$ on the piston line EQF ($n = (n_1, n_2)$ is the normal to the piston line). The known potential φ_0 is given by the equation

$$\begin{gathered} \varphi_0 = -{}^1/_2\eta_1^2 - {}^1/_2 C^2 + C^2(1-\gamma)^{-1}, \qquad \xi_1 \in M_1 \\ \varphi_0 = -{}^1/_2\eta_1^2 - {}^1/_2(U_0\cos\alpha - \xi_1)^2 + C_1^2(1-\gamma)^{-1}, \qquad \xi_1 \in M_2 \end{gathered} \tag{1.10}$$

In the exact formulation the problem is complicated by the fact that the quasi-linear equation (1.3) is of the mixed type and region BEQF contains regions of both hyperbolicity and ellipticity of the equation.

Considering the case of small angles α, we set

$$\varphi = \psi_0 + \alpha\psi \qquad (\psi_0 = \varphi_0|_{\alpha=0}) \tag{1.11}$$

Here, ψ_0 is the potential of the undisturbed motion and ψ is the unknown potential of the disburbances. Substituting (1.11) into (1.3) and discarding terms containing the second and higher powers of α, we obtain the equations for ψ

$$\begin{gathered} \left(\frac{2}{\gamma+1} C_0 + \frac{\gamma-1}{\gamma+1}\xi\right)\eta\psi_{\xi\eta} + \frac{1}{2}\left(\eta^2 - \left(\frac{2}{\gamma+1} C_0 + \frac{\gamma-1}{\gamma+1}\xi\right)^2\right)\psi_{\eta\eta} \\ -\left(\frac{2}{\gamma+1} C_0 + \frac{\gamma-1}{\gamma+1}\xi\right)\psi_\xi - \frac{\gamma-1}{\gamma+1}\eta\psi_\eta + \frac{\gamma-1}{\gamma+1}\psi = 0 \\ U_0 + C_1 \leqslant \xi \leqslant C_0 \end{gathered} \tag{1.12}$$

$$\begin{gathered} ((U_0 - \xi)^2 - C_1^2)\,\psi_{\xi\xi} - 2(U_0 - \xi)\,\eta\psi_{\xi\eta} + (\eta^2 - C_1^2)\,\psi_{\eta\eta} = 0 \\ U_0 \leqslant \xi \leqslant U_0 + C_1 \end{gathered} \tag{1.13}$$

Equation (1.13) is of the mixed type, its characteristics in the region of hyperbolicity are tangents to the line of degeneracy given by the equation

$$(\xi - U_0)^2 + \eta^2 = C_1^2 \tag{1.14}$$

Here and in what follows C_1 is the speed of sound in the constant flow at $\alpha = 0$ (see (1.6). The characteristic MH (Fig. 2) of Eq. (1.12) and (1.13) corresponding to BE is given by Eqs. (1.8) and (1.9) with $\alpha = 0$. Transfer of the conditions from characteristic BE to characteristic MH and the impermeability condition from line EQ to HA leads to the following problem for Eqs. (1.12) and (1.13): to find the solution ψ satisfying the conditions

$$\begin{aligned} \psi|_{MN} &= 2(\gamma+1)^{-1}(C_0 - \xi)\eta, \quad && U_0 + C_1 \leqslant \xi \leqslant C_0 \\ \psi|_{MH} &= -U_0\eta, && U_0 \leqslant \xi \leqslant U_0 + C_1 \end{aligned} \tag{1.15}$$

and the condition $\psi_\xi = 0$ at $\xi = U_0$. The region of definition of the solution is divided into four subregions. In region 1, bounded by the characteristic MN and the segment ML of the ξ -axis, the mixed problem for Eq. (1.12) is solved. After solving this problem we obtain a Goursat problem in region 2 bounded by the characteristics LN and NK (NH) (Fig. 2a and b). In region 3 the mixed problem for Eq. (1.13) is solved, and then, when the solution has been found in the regions of hyperbolicity 1, 2, and 3, the mixed boundary value problem for (1.13) is solved in region 4, the function ψ being given on a line of degeneracy of the (1.14) type, which enters into the boundary of the region. The form of region 3 varies with the piston velocity U_0 and the adiabatic exponent γ. At

$$U_0 = U_* = \frac{2C_0}{\gamma-1}\left(\left(2\,\frac{\gamma-1}{\gamma+1}\right)^{\frac{\gamma-1}{3-\gamma}} - 1\right)$$

region 3 degenerates into a point, since the characteristic NH is perpendicular to the line $\xi = U_0$. At $U_0 < U_*$ region 3 is bounded by a characteristic of the second family starting from the point H and by the line $\xi = U_0$. In what follows we distinguish three cases: $U_0 > U_*$ (Fig. 2a), b) $U_0 < U_*$ (Fig. 2b), c) $U_0 = U_*$.

2. Solution in Region 1

If Eq. (1.12) is reduced to the characteristic variables

$$\begin{aligned} z &= \eta^2\left(\frac{2}{\gamma+1}C_0 + \frac{\gamma-1}{\gamma+1}\xi\right)^{-\frac{\gamma+1}{\gamma-1}} + \frac{\gamma+1}{\gamma-3}\left(\frac{2}{\gamma+1}C_0 + \frac{\gamma-1}{\gamma+1}\xi\right)^{\frac{\gamma-3}{\gamma-1}} \\ \tau &= -\frac{\gamma+1}{\gamma-3}\left(\frac{2}{\gamma+1}C_0 + \frac{\gamma-1}{\gamma+1}\xi\right)^{\frac{\gamma-3}{\gamma-1}} \end{aligned} \tag{2.1}$$

then the equation obtained

$$\psi_{z\tau} - \frac{1}{\gamma-3}\,\frac{(\gamma-1)z + 2\tau}{\tau(z+\tau)}\psi_z - \frac{1}{2(z+\tau)}\psi_\tau + \frac{\gamma-1}{\gamma-3}\,\frac{1}{2\tau(z+\tau)}\psi = 0 \tag{2.2}$$

can be integrated in quadratures. Setting

$$g = \psi_z - \frac{1}{2(z+\tau)}\psi \tag{2.3}$$

we obtain the ordinary differential equation for g:

$$g_\tau - \frac{1}{\gamma-3}\,\frac{(\gamma-1)z + 2\tau}{(z+\tau)\tau}\,g = 0 \tag{2.4}$$

Solving Eq. (2.3) and (2.4) successively, we obtain the general solution of Eq. (1.12):

$$\psi = \left(\left(\frac{3-\gamma}{\gamma+1}\tau\right)^{\frac{(\gamma-1)}{(\gamma-3)}}\int_{a_1}^{z}\frac{f(\zeta)\,d\zeta}{(\zeta+\tau)^{3/2}} + h(\tau)\right)\sqrt{z+\tau} \tag{2.5}$$

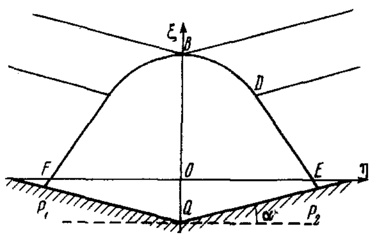

Fig. 1

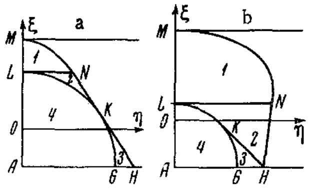

Fig. 2

Here, f and h are arbitrary functions; the variables z and τ are expressed in terms of ξ and η in accordance with (2.1). The characteristic MN is given by the equation

$$z = a_0 = \frac{\gamma+1}{\gamma-3} C_0^{\frac{(\gamma-3)}{(\gamma-1)}}$$

Therefore in (2.5) it is convenient to take the constant $a_1 = a_0$. Satisfying condition (1.15) on MN, we determine

$$h(\tau) = \frac{2}{\gamma-1}\left(C_0 - \left(\frac{3-\gamma}{\gamma+1}\tau\right)^{\frac{(\gamma-1)}{(\gamma-3)}}\right)\left(\frac{3-\gamma}{\gamma+1}\tau\right)^{-1/2} \tag{2.6}$$

From the symmetry condition $\psi_\eta = 0$ at $\eta = 0$ we obtain the Abel integral equation for f:

$$\int_{a_0}^{-\tau} \frac{f'(\zeta)\, d\zeta}{(\zeta+\tau)^{1/2}} = -\frac{1}{\gamma-1}\left(C_0 - \left(\frac{3-\gamma}{\gamma+1}\tau\right)^{\frac{(\gamma-1)}{(\gamma-3)}}\right)\left(\frac{3-\gamma}{\gamma+1}\tau\right)^{-1/2} \tag{2.7}$$

Inverting the integral operator on the right-hand side of (2.7) and substituting the result and the expression for h(τ) (2.6) into (2.5), we obtain the form of ψ in region 1:

$$\psi = -\frac{2}{\pi(\gamma-1)}\left(\frac{3-\gamma}{\gamma+1}\tau\right)^{\frac{(\gamma-1)}{(\gamma-3)}} \int_{a_0}^{z} \frac{\sqrt{\zeta - z}}{\tau+\zeta}\left(C_0 - \left(\frac{\gamma-3}{\gamma+1}\zeta\right)^{\frac{(\gamma-1)}{(\gamma-3)}}\right)\left(\frac{\gamma-3}{\gamma+1}\right)^{-1/2} d\zeta \tag{2.8}$$

$$+ \frac{2\sqrt{z+\tau}}{\gamma-1}\left(C_0 - \left(\frac{3-\gamma}{\gamma+1}\tau\right)^{\frac{(\gamma-1)}{(\gamma-3)}}\right)\left(\frac{3-\gamma}{\gamma+1}\tau\right)^{-1/2}$$

Then, using the value obtained for the potential ψ on characteristic LN, we solve the problem in region 2. We denote

$$\psi_1(\eta / C_1) = \psi(U_0 + C_1, \eta) \tag{2.9}$$

3. Solution of the Problem in Region 2

The data on characteristic MH have a discontinuity of the first derivative at point N, as follows from (1.15). Therefore, along characteristic LN after solution of the Goursat problem in region 2 the derivative of φ_ξ will have a discontinuity of order α.

We go over to polar coordinates in Eq. (1.13):

$$\xi - U_0 = r\cos\theta, \qquad \eta = r\sin\theta$$
$$r^2(r^2 - C_1^2)\psi_{rr} - C_1^2\psi_{\theta\theta} - C_1^2\, r\psi_r = 0 \tag{3.1}$$

This equation admits a three-parameter group of transformations and, as follows from the results of [3], reduces to the Euler-Poisson equation. Applying the algorithm described in [3], we make the following substitutions:

$$s = \operatorname{tg}\frac{\theta + \arccos(C_1/r)}{2}, \quad q = \operatorname{tg}\frac{\theta - \arccos(C_1/r)}{2}, \quad \psi = \frac{\chi}{sq+1}$$
$$0 \leqslant \arccos(C_1/r) < {}^1\!/_2\pi \tag{3.2}$$

Equation (3.2) reduces to the Euler-Poisson equation

$$\chi_{sq} + \frac{1}{s-q}(\chi_s - \chi_q) = 0 \tag{3.3}$$

The general solution of Eq. (3.3) is easily written out, and from (3.2) we obtain the general solution of Eq. (1.13):

$$\psi = \frac{s-q}{sq+1}(\omega'(q) + \omega_1'(s)) + \frac{2}{sq+1}(\omega(q) - \omega_1(s)) \tag{3.4}$$

Here, ω and ω_1 are arbitrary functions and the variables s and q are expressed in terms of ξ and η.

Knowing the general solution, we can solve the Goursat problem in region 2. In the sq plane the characteristics LN and NK(MH), on which the data are given, are given by the equations q = 0 and

$$s = s_0 = \left(\frac{\gamma+1}{3-\gamma} - \frac{\gamma+1}{3-\gamma} C_0^{\frac{(\gamma-3)}{(\gamma-1)}} C_1^{-\frac{(\gamma-3)}{(\gamma-1)}}\right)^{1/2}$$

respectively. On these characteristics the function ψ is known:

$$\psi|_{s=s_0} = -U_0\eta = -U_0 C_1 \frac{s_0+q}{s_0 q+1}, \quad \psi|_{q=0} = \psi_1(s)$$

From the data on the characteristics the functions ω and ω_1 are determined as the solutions of ordinary linear differential equations. Let us determine the arbitrariness in the determination of ω and ω_1. If

$$\omega(q) = b_0 q^2 + b_1 q + b_2, \qquad \omega_1(s) = b_0 s^2 + b_1 s + b_2$$

then the function $\psi = 0$. Consequently, without loss of generality, we may assume that

$$\omega(0) = 0, \quad \omega'(0) = 0, \quad \omega_1(s_0) = 0$$

Satisfying the conditions on the characteristics, we determine ω and ω_1:

$$\omega_1(s) = s^2 \int_{s_0}^{s} \psi_1(\zeta)\zeta^{-3} d\zeta, \qquad \omega(q) = -\frac{U_0 C_1}{s_0} q^2 \tag{3.5}$$

As follows from the first of Eq. (3.5) and expression (2.9) for ψ_1, the derivative $\omega_1''(s) = 0$ (ln s); therefore the derivative of ψ_r is unbounded at point L, and $\psi_r = 0$ (ln s); however, the derivative of ψ_θ is bounded everywhere in region 2. The unboundedness of the derivatives of the solution of the Goursat problem at the points of tangency of the characteristics and the line of degeneracy is the known fact in the theory of equations of mixed type.

Then, in case c it is possible to proceed to solve the problem in region 4; in cases a and b the problem in region 3 is solved.

4. Solution of the Problem in Region 3

Case a. Here we encounter the following mixed problem: the function ψ is given on characteristic KH and $\psi_\xi = 0$ at $\xi = U_0$. In variables s and q these conditions take the form

$$\psi|_{q=s_0} = -U_0 C_1 \frac{s+s_0}{ss_0+1}, \quad s\psi_s + q\psi_q|_{sq=1} = 0 \tag{4.1}$$

Without loss of generality, we may assume

$$\omega(s_0) = 0, \quad \omega'(s_0) = 0, \quad \omega_1(1/s_0) = U_0 C_1 / s_0$$

From the condition on the characteristic KH we determine

$$\omega_1(s) = U_0 C_1 s \tag{4.2}$$

Then, satisfying the second of conditions (4.1) on line GH

$$\tfrac{1}{2}(1-q^2)\omega''(q) + q\omega'(q) - \omega(q) = 0$$

we determine $\omega(q) = 0$. Thus, the potential ψ in region 3 is given by the equation

$$\psi = -U_0\eta \tag{4.3}$$

Case b. The data on characteristic KH, given by the equation $q = q_1 = 1/s_0$, are obtained after solving the problem in region 2. Let the functions ω_1^0 and ω^0 be obtained in solving the problem in region 2. Without loss of generality, we may assume that the functions ω and ω_1 in region 3 are such that

$$\omega(q_1) = \omega^0(q_1) = -U_0 C_1 q_1^3, \qquad \omega'(q_1) = \omega^{0\prime}(q_1) = -2U_0 C_1 q_1^2$$
$$\omega_1(s_0) = \omega_1^0(s_0) = 0 \tag{4.4}$$

Since the function ω_1^0 satisfies the equation along the characteristic, from which ω_1 must be determined,by virtue of the uniqueness theorem for linear ordinary equations

$$\omega_1(s) = \omega_1^0(s) = s^2 \int_{s_0}^{s} \psi_1(\zeta)\, \zeta^{-3}\, d\zeta \tag{4.5}$$

The second of conditions (4.1) gives a second-order equation for determining $\omega(q)$:

$$\frac{1}{2}(1-q^2)\,\omega''(q) + q\omega'(q) - \omega(q) = \frac{q^2-1}{2q^2}\,\omega_1''\left(\frac{1}{q}\right) + \frac{1}{q}\,\omega_1'\left(\frac{1}{q}\right) - \omega_1\left(\frac{1}{q}\right)$$

The general solution of the corresponding homogeneous equation has the form

$$\omega = D_1 q + D_2(q^2 - 1)$$

The function

$$\omega = -q^2\, \omega_1(1/q)$$

is a particular solution of the inhomogeneous equation.

Determining the constants D_1 and D_2 from (4.4), we find

$$\omega(q) = U_0 C_1 q - U_0 C_1 q_1 (q^2+1) - \int_{s_0}^{1/q} \psi_1(\zeta)\, \zeta^{-3}\, d\zeta \tag{4.6}$$

Thus, the solution of the problem has been found everywhere in the regions of hyperbolicity of the linearized equations.

5. Solution of the Problem in Region 4

After solving the problems in the region of hyperbolicity of Eq. (1.13) we must turn to the mixed boundary value problem in the region of ellipticity. The potential ψ can be determined in region 4 if it is known on the semicircle (1.14), and if the derivative $\xi = U_0$ is given on the diameter $\psi_\xi = 0$. As a result of the uniqueness of the solution of this problem it is sufficient to solve the Dirichlet problem in a circle with Dirichlet data continued evenly onto the lower semicircle. Here, too, it is convenient to go over to the coordinates s, k, which will be complex in region 4:

$$s = \bar{q}, \qquad s = \lambda + i\mu$$

If χ is the solution of Eq. (3.3), then the potential ψ in region 4 is related with the function χ by the expression

$$\psi = \mathrm{Re}\,\frac{\chi}{sq+1}$$

Let $\sigma = \mathrm{Re}\ \chi$; then from (3.3) there follows the equation for σ

$$\Delta\sigma - \frac{2}{\mu}\,\sigma_\mu = 0 \qquad \left(\Delta = \frac{\partial^2}{\partial\lambda^2} + \frac{\partial^2}{\partial\mu^2}\right) \tag{5.1}$$

The solution of this equation is expressed in terms of the arbitrary harmonic function F of the variables λ and μ:

$$\sigma = F - \mu F_\mu$$

As a result of transformation (3.2) the circle $(\xi - U_0)^2 + \eta^2 \le C_1^2$ in the plane $\xi\eta$ goes over into the half-plane $\mu \ge 0$. The circle (1.14) is mapped into the axis $\mu = 0$ so that the upper semicircle goes over into the segment $[-1, 1]$ of the λ-axis. The continuation of the Dirichlet data

$$\psi|_{\mu=0} = f_0(\lambda), \qquad |f_0(\lambda)| < \infty$$

onto the entire axis is realized in accordance with the equation

$$f_0(\lambda) = f_0(\lambda^{-1}), \qquad |\lambda| \geqslant 1$$

The potential ψ is expressed in terms of the harmonic function F

$$\psi = \frac{1}{1+\lambda^2+\mu^2}(F - \mu F_\mu) \tag{5.2}$$

Therefore the starting boundary value problem reduces to the following problem for the function F harmonic in the half-plane $\mu \geq 0$

$$F|_{\mu=0} = (1+\lambda^2) f_0(\lambda), \quad |F| \leqslant A_1^2 |s|^2, \quad |s| \to \infty \quad (A_1^2 = \text{const}) \tag{5.3}$$

The latter inequality follows from the requirement that ψ be bounded. The data on the axis $\mu = 0$ are expressed in terms of the functions ω and ω_1 obtained in solving the problems in regions 2 and 3, because

$$f_0(\lambda) = \frac{2}{\lambda^2+1}(\omega(\lambda) - \omega_1(\lambda)) \qquad (0 \leqslant \lambda \leqslant 1), \qquad f_0(\lambda) = f_0(-\lambda), \qquad f_0(\lambda) = f_0(\lambda^{-1}) \tag{5.4}$$

It follows from (5.4) and (3.5) that $(1+\lambda^2) f_0(\lambda)$ can be represented as follows:

$$(1+\lambda^2) f_0(\lambda) = f_1(\lambda) + \lambda^2 \psi_1(0) + {}^1/_2 \psi_1''(0) \ln(\lambda^2+1)$$

Here, $f_1(\lambda)$ is a function bounded on the entire λ-axis.

The solution of problem (5.3) is the function

$$F(\lambda, \mu) = \frac{\mu}{\pi} \int_{-\infty}^{\infty} \frac{f_1(\zeta)\, d\zeta}{(\lambda-\zeta)^2+\mu^2} + \psi_1(0)(\lambda^2 - \mu^2) + \frac{1}{2}\psi_1''(0) \ln(\lambda^2 + (\mu+1)^2) \tag{5.5}$$

As it is easy to establish, this solution is unique correct to unimportant terms of the form $b\mu + d\lambda\mu$.

Thus, the linear problem has been completely solved. The equations obtained make it possible to calculate the physical quantities u, v, and c in the double-wave region. At large absolute piston velocities U_0 a vacuum zone is formed in the neighborhood of the vertex of the piston angle Q. Then in regions 3 and 4 the solution obtained will not be applicable, since in this case it would be necessary to solve the problem with a free boundary. The critical piston velocity U_0 at which separation of the gas first takes place is determined from the equation

$$C_1^2 + \alpha(1-\gamma)\psi(v_0, 0) = 0$$

At lesser velocities the solution obtained describes the motion of the gas in first approximation.

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